Stability analysis of BAM neural networks with time-varying delays*

ZHANG Huaguang[1]** and WANG Zhanshan[1,2]

(1. School of Information Science and Engineering, Northeastern University, Shenyang 110004, China; 2. Department of Information Engineering, Shenyang Ligong University, Shenyang 110168, China)

Accepted on April 30, 2006

Abstract Some new criteria for the global asymptotic stability of the equilibrium point for the bi-directional associative memory neural networks with time varying delays are presented. The obtained results present the structure of linear matrix inequality which can be solved efficiently. The comparison with some previously reported results in the literature demonstrates that the results in this paper provide one more set of criteria for determining the stability of the bi-directional associative memory neural networks with delays.

Keywords: bi-directional associative memory (BAM) neural networks, time varying delays, global asymptotic stability, linear matrix inequality (LMI).

Neural networks have been extensively studied in the past decade and have been used in various applications such as designing associative memories and solving optimization problems. Recently, many researchers have studied the stability properties of neural networks and presented various sufficient conditions for the globally asymptotic stability of the equilibrium point of different classes of neural networks. On the other hand, the delayed versions of neural networks have also been proved to be important for solving some classes of motion-related optimization problems. Some results concerning the dynamical behavior of neural networks with delays have been reported[1—5]. In some of the recent research papers, researchers have paid particular attention to the stability analysis of bi-directional associative memory neural networks with time delays as this kind of neural networks has been shown to be a useful network model for applications in pattern recognition, solving optimization problems and so on[3—11]. Some results of the global asymptotic/exponential stability of the equilibrium point for bi-directional associative memory (BAM) neural networks with constant delays can be found in Refs. [3—9] which obtain some algebraic inequalities. Although the suitability of the results in Refs. [4—9] is improved by involving many unknown parameters to be tuned, we have no systematic approach to adjust those parameters in advance, and correspondingly it is difficult to check the validness of those results. Moreover, the results in Refs. [3—9] ignore the sign difference of entries in connection matrix, and the effects of neuron excitatory and inhibitory on the neural networks are not considered. At present, linear matrix inequality (LMI) technique has been used to tackle the stability problem of neural networks, and the obtained results can be easy to check and can eliminate the difference between neuron excitatory and inhibitory on neural networks[1,2]. Therefore, in this paper, we will present some new sufficient conditions for the global asymptotic stability of the BAM neural networks with multiple time varying delays using LMI technique.

1 Problem description and preliminaries

Consider the following BAM neural networks with multiple time varying delays:

$$\begin{aligned} \dot{u}_i(t) &= -a_i u_i(t) + \sum_{j=1}^{m} b_{ij}\bar{g}_j(v_j(t-\tau_{ij}(t))) + U_i, \\ &\qquad i = 1,\cdots,n, \\ \dot{v}_i(t) &= -c_i v_i(t) + \sum_{j=1}^{n} d_{ij}\bar{f}_j(u_j(t-z_{ij}(t))) + J_i, \\ &\qquad i = 1,\cdots,m, \end{aligned} \tag{1}$$

where u_i, v_j are the states of neurons; $a_i > 0$ and c_j

* Supported by National Natural Science Foundation of China (Grant Nos. 60534010, 60572070, 60521003) and the Program for Changjiang Scholars and Innovative Research Team in University

** To whom correspondence should be addressed. E-mail: hg_zhang@21cn.com

>0 denote the neuron charging time constants and passive decay rate, respectively; b_{ij}, d_{ji} are synaptic connection strengths; $\bar{g}_j(\cdot)$ and $\bar{f}_i(\cdot)$ represent the activation functions of the neurons; U_i, J_j are the external constant inputs; $\tau_{ij}(t)\geqslant 0$, $z_{ji}(t)\geqslant 0$, $\dot{\tau}_{ij}(t)$ and $\dot{z}_{ji}(t)$ denote the change rate of $\tau_{ij}(t)$ and $z_{ji}(t)$, respectively, $i=1,\cdots,n, j=1,\cdots,m$.

Assumption 1. The bounded activation functions $\bar{g}_j(\cdot),\bar{f}_k(\cdot)$ satisfy the following conditions

$$0\leqslant\frac{\bar{g}_j(\zeta)-\bar{g}_j(\xi)}{\zeta-\xi}\leqslant\delta_j^g,$$
$$0\leqslant\frac{\bar{f}_k(\zeta)-\bar{f}_k(\xi)}{\zeta-\xi}\leqslant\delta_k^f,$$

for $\zeta,\xi\in\mathscr{R}$, $\zeta\neq\xi$ and for some $\delta_j^g>0,\delta_k^f>0,k=1,\cdots,n,j=1,\cdots,m$.

Let $\boldsymbol{\Delta}_g=\mathrm{diag}(\delta_1^g,\cdots,\delta_m^g)\in\mathscr{R}^{m\times m}$, $\boldsymbol{\Delta}_f=\mathrm{diag}(\delta_1^f,\cdots,\delta_n^f)\in\mathscr{R}^{n\times n}$. Obviously, positive diagonal matrices $\boldsymbol{\Delta}_g$ and $\boldsymbol{\Delta}_f$ are all non-singular.

Lemma 1[2]. For two vectors $\boldsymbol{X}$ and $\boldsymbol{Y}$ with compatible dimensions, and two matrices $\boldsymbol{P}$, $\boldsymbol{Q}=\boldsymbol{Q}^{\mathrm{T}}>0$ with compatible dimensions, the following inequality holds

$$-\boldsymbol{X}^{\mathrm{T}}\boldsymbol{Q}\boldsymbol{X}+2\boldsymbol{X}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{Y}\leqslant\boldsymbol{Y}^{\mathrm{T}}\boldsymbol{P}^{\mathrm{T}}\boldsymbol{Q}^{-1}\boldsymbol{P}\boldsymbol{Y}.$$

To proceed conveniently, let $\boldsymbol{I}$ denote an identity matrix with compatible dimension, let $\boldsymbol{u}^*=(u_1^*,\cdots,u_n^*)^{\mathrm{T}}$ and $\boldsymbol{v}^*=(v_1^*,\cdots,v_m^*)^{\mathrm{T}}$ denote the equilibrium point of model (1), $\boldsymbol{B}=(b_{ij})_{n\times m}$, $\boldsymbol{D}=(d_{ij})_{m\times n}$, $\boldsymbol{A}=\mathrm{diag}(a_1,\cdots,a_n)$, $\boldsymbol{C}=\mathrm{diag}(c_1,\cdots,c_m)$; $\boldsymbol{B}_i\in\mathscr{R}^{n\times m}$, whose ith row is composed by the ith row of $\boldsymbol{B}$, and the other rows are zeros; $\boldsymbol{D}_j\in\mathscr{R}^{m\times n}$, whose jth row is composed by the jth row of $\boldsymbol{D}$, and the other rows are zeros, $i=1,\cdots,n,j=1,\cdots,m$.

Let $\boldsymbol{x}(t)=\boldsymbol{u}(t)-\boldsymbol{u}^*$ and $\boldsymbol{y}(t)=\boldsymbol{v}(t)-\boldsymbol{v}^*$. Then model (1) is transformed to the following form:

$$\dot{x}_i(t)=-a_ix_i(t)+\sum_{j=1}^m b_{ij}g_j(y_j(t-\tau_{ij}(t))),\quad i=1,\cdots,n,$$
$$\dot{y}_i(t)=-c_iy_i(t)+\sum_{j=1}^n d_{ij}f_j(x_j(t-z_{ij}(t))),\quad i=1,\cdots,m,\tag{2}$$

or

$$\dot{\boldsymbol{x}}(t)=-\boldsymbol{A}\boldsymbol{x}(t)+\sum_{j=1}^n\boldsymbol{B}_j\boldsymbol{g}(\boldsymbol{y}(t-\boldsymbol{\tau}_j(t))),$$
$$\dot{\boldsymbol{y}}(t)=-\boldsymbol{C}\boldsymbol{y}(t)+\sum_{j=1}^m\boldsymbol{D}_j\boldsymbol{f}(\boldsymbol{x}(t-\boldsymbol{z}_j(t))),\tag{3}$$

where

$$\boldsymbol{g}(\boldsymbol{y}(t-\boldsymbol{\tau}_j(t)))=(g_1(y_1(t-\tau_{j1}(t))),\cdots,g_m(y_m(t-\tau_{jm}(t))))^{\mathrm{T}},$$
$$\boldsymbol{f}(\boldsymbol{x}(t-\boldsymbol{z}_k(t)))=(f_1(x_1(t-z_{k1}(t))),\cdots,f_n(y_n(t-\tau_{kn}(t))))^{\mathrm{T}},$$
$$g_i(y_i(t-\tau_{ji}(t)))=\bar{g}_i(y_i(t-\tau_{ji}(t))+v_i^*)-\bar{g}_i(v_i^*),$$
$$f_k(x_k(t-z_{jk}(t)))=\bar{f}_k(x_k(t-z_{jk}(t))+u_k^*)-\bar{f}_k(u_k^*),$$
$$j=1,\cdots,n,\ k=1,\cdots,m.$$

Clearly, activation functions $g_j(\cdot)$ and $f_k(\cdot)$ also satisfy Assumption 1.

Theorem 1. Suppose that $\dot{\tau}_{ij}(t)<1$ and $\dot{z}_{ji}(t)<1$. If the following linear matrix inequalities

$$\begin{bmatrix}-2\boldsymbol{P}\boldsymbol{A}+\sum_{i=1}^m\boldsymbol{\Delta}_f\boldsymbol{F}_i\boldsymbol{\Delta}_f & \boldsymbol{P}\boldsymbol{B}_1 & \cdots & \boldsymbol{P}\boldsymbol{B}_n\\ \boldsymbol{B}_1^{\mathrm{T}}\boldsymbol{P} & -\eta_1^1\boldsymbol{E}_1 & \cdots & \boldsymbol{0}\\ \vdots & \vdots & \ddots & \vdots\\ \boldsymbol{B}_n^{\mathrm{T}}\boldsymbol{P} & \boldsymbol{0} & \cdots & -\eta_n^1\boldsymbol{E}_n\end{bmatrix}<\boldsymbol{0},$$
$$\begin{bmatrix}-2\boldsymbol{Q}\boldsymbol{C}+\sum_{i=1}^n\boldsymbol{\Delta}_g\boldsymbol{E}_i\boldsymbol{\Delta}_g & \boldsymbol{Q}\boldsymbol{D}_1 & \cdots & \boldsymbol{Q}\boldsymbol{D}_n\\ \boldsymbol{D}_1^{\mathrm{T}}\boldsymbol{Q} & -\eta_1^2\boldsymbol{F}_1 & \cdots & \boldsymbol{0}\\ \vdots & \vdots & \ddots & \vdots\\ \boldsymbol{D}_n^{\mathrm{T}}\boldsymbol{Q} & \boldsymbol{0} & \cdots & -\eta_m^2\boldsymbol{F}_m\end{bmatrix}<\boldsymbol{0}\tag{4}$$

have positive definite symmetric matrices $\boldsymbol{P}$, $\boldsymbol{Q}$, positive diagonal matrices $\boldsymbol{E}_1,\cdots,\boldsymbol{E}_n,\boldsymbol{F}_1,\cdots,\boldsymbol{F}_m$, then the origin of system (3) is globally asymptotically stable, and is independent of the magnitude of time varying delays, where

$$\eta_i^1=\min(1-\dot{\tau}_{ij}(t)),\quad\eta_j^2=\min(1-\dot{z}_{ji}(t)),$$
$$\boldsymbol{F}_j=\mathrm{diag}(f_{1j},\cdots,f_{nj}),\quad\boldsymbol{E}_i=\mathrm{diag}(e_{i1},\cdots,e_{im}),$$
$$i=1,\cdots,n,\ j=1,\cdots,m.$$

Proof. Consider the following Lyapunov-Krasovskii functional:

$$V(\boldsymbol{x}(t),\boldsymbol{y}(t))=\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{P}\boldsymbol{x}(t)+\boldsymbol{y}^{\mathrm{T}}(t)\boldsymbol{Q}\boldsymbol{y}(t)+\sum_{i=1}^n\sum_{j=1}^m\int_{t-\tau_{ij}(t)}^t e_{ij}g_j^2(y_j(s))\mathrm{d}s+\sum_{i=1}^m\sum_{j=1}^n\int_{t-z_{ij}(t)}^t f_{ij}f_j^2(x_j(s))\mathrm{d}s.\tag{5}$$

The derivative of (5) along the trajectories of (3) is

$$\begin{aligned}\dot V(x(t),y(t)) \leqslant & -2x^{\mathrm T}(t)PAx(t)+2x^{\mathrm T}(t)\\ & \cdot P\sum_{j=1}^{n}B_jg(y(t-\tau_j(t)))\\ & -2y^{\mathrm T}(t)QCy(t)+2y^{\mathrm T}(t)\\ & \cdot Q\sum_{j=1}^{m}D_jf(x(t-z_j(t)))\\ & +\sum_{i=1}^{n}[g^{\mathrm T}(y(t))E_ig(y(t))\\ & -\eta_i^1g^{\mathrm T}(y(t-\tau_i(t)))E_ig(y(t-\tau_i(t)))]\\ & +\sum_{i=1}^{m}[f^{\mathrm T}(x(t))F_if(x(t))\\ & -\eta_i^2f^{\mathrm T}(x(t-z_i(t)))F_if(x(t-z_i(t)))].\end{aligned}\tag{6}$$

By Lemma 1 and Assumption 1, the following inequalities hold,

$$\begin{aligned}& 2x^{\mathrm T}(t)P\sum_{i=1}^{n}B_ig(y(t-\tau_i(t)))\\ & \quad -\sum_{i=1}^{n}\eta_i^1g^{\mathrm T}(y(t-\tau_i(t)))\\ & \quad \cdot E_ig(y(t-\tau_i(t)))\\ & \leqslant \sum_{i=1}^{n}\frac{1}{\eta_i^1}x^{\mathrm T}(t)PB_iE_i^{-1}B_i^{\mathrm T}Px(t),\end{aligned}\tag{7}$$

$$\begin{aligned}& 2y^{\mathrm T}(t)Q\sum_{i=1}^{m}D_if(x(t-z_i(t)))\\ & \quad -\sum_{i=1}^{m}\eta_i^2f^{\mathrm T}(x(t-z_i(t)))\\ & \quad \cdot F_if(x(t-z_i(t)))\\ & \leqslant \sum_{i=1}^{m}\frac{1}{\eta_i^2}y^{\mathrm T}(t)QD_iF_i^{-1}D_i^{\mathrm T}Qy(t),\end{aligned}\tag{8}$$

$$\sum_{i=1}^{n}g^{\mathrm T}(y(t))E_ig(y(t)) \leqslant \sum_{i=1}^{n}y^{\mathrm T}(t)\Delta_gE_i\Delta_gy(t),\tag{9}$$

$$\sum_{i=1}^{m}f^{\mathrm T}(x(t))F_if(x(t)) \leqslant \sum_{j=1}^{m}x^{\mathrm T}(t)\Delta_fF_i\Delta_fx(t).\tag{10}$$

Substituting (7)—(10) into (6) yields

$$\begin{aligned}\dot V(x(t),y(t)) \leqslant & x^{\mathrm T}(t)\Big[-2PA\\ & +\sum_{i=1}^{n}\frac{1}{\eta_i^1}PB_iE_i^{-1}B_i^{\mathrm T}P+\sum_{i=1}^{m}\Delta_fF_i\Delta_f\Big]x(t)\\ & +y^{\mathrm T}(t)\Big[-2QC+\sum_{i=1}^{m}\frac{1}{\eta_i^2}QD_iF_i^{-1}D_i^{\mathrm T}Q\\ & +\sum_{i=1}^{n}\Delta_gE_i\Delta_g\Big]y(t).\end{aligned}\tag{11}$$

If the following inequalities hold,

$$-2QC+\sum_{i=1}^{m}\frac{1}{\eta_i^2}QD_iF_i^{-1}D_i^{\mathrm T}Q+\sum_{i=1}^{n}\Delta_gE_i\Delta_g<0,\tag{12}$$

$$-2PA+\sum_{i=1}^{n}\frac{1}{\eta_i^1}PB_iE_i^{-1}B_i^{\mathrm T}P+\sum_{i=1}^{m}\Delta_fF_i\Delta_f<0,\tag{13}$$

then for any $x(t)\neq 0$ or $y(t)\neq 0$, $\dot V(x(t),y(t))<0$. $\dot V(x(t),y(t))=0$ only and if only $x(t)=y(t)=0$. Since $V(x(t),y(t))$ is radically unbounded, then by Lyapunov stability theory, the origin of system (3), or equivalently the equilibrium point of system (1), is globally asymptotically stable, and independent of the magnitude of time varying delays. By Schur complement[11], conditions (12) and (13) are equivalent to conditions (4). The proof of Theorem 1 is completed.

When $\tau_{ij}(t)=\tau_j(t)$ and $z_{ij}(t)=z_j(t)$ in (2), (3) is replaced by the following expression,

$$\begin{aligned}\dot x(t) &= -Ax(t)+Bg(y(t-\tau(t))),\\ \dot y(t) &= -Cy(t)+Df(x(t-z(t))),\end{aligned}\tag{14}$$

where

$$\tau(t)=(\tau_1(t),\cdots,\tau_m(t))^{\mathrm T}\in\mathscr{R}^{m\times 1},$$

$$z(t)=(z_1(t),\cdots,z_n(t))^{\mathrm T}\in\mathscr{R}^{n\times 1},$$

$$B=(b_{ij})_{n\times m},\quad D=(d_{ij})_{m\times n}.$$

Now, we will obtain the following stability results for system (14).

Corollary 1. Suppose that $\dot\tau_j(t)<1$ and $\dot z_j(t)<1$ in (14). If the following matrix inequalities

$$\begin{aligned}& -2PA+\frac{1}{\eta^1}PBE^{-1}B^{\mathrm T}P+\Delta_fF\Delta_f<0,\\ & -2QC+\frac{1}{\eta^2}QDF^{-1}D^{\mathrm T}Q+\Delta_gE\Delta_g<0,\end{aligned}\tag{15}$$

have positive definite symmetric matrices P, Q and positive diagonal matrices E, F, then the origin of system (14) is globally asymptotically stable, and is independent of the magnitude of time varying delays, where

$$\eta^1=\min(1-\dot\tau_j(t)),\quad \eta^2=\min(1-\dot z_i(t)),$$
$$i=1,\cdots,n,\ j=1,\cdots,m.$$

Proof. Consider the following Lyapunov-Krasovskii functional,

$$V(x(t),y(t))=x^{\mathrm T}(t)Px(t)+y^{\mathrm T}(t)Qy(t)$$
$$+\sum_{j=1}^{m}\int_{t-\tau_j(t)}^{t}e_jg_j^2(y_j(s))\mathrm{d}s$$

$$+\sum_{j=1}^{n}\int_{t-z_j(t)}^{t} f_j f_j^2(x_j(s))\mathrm{d}s. \quad (16)$$

The derivative of (16) along the trajectories of (14) is

$$\begin{aligned}\dot V(x(t),y(t)) \leqslant & -2x^{\mathrm T}(t)PAx(t)\\ &+2x^{\mathrm T}(t)PBg(y(t-\tau(t)))\\ &-2y^{\mathrm T}(t)QCy(t)+2y^{\mathrm T}(t)QDf(x(t-z(t)))\\ &+g^{\mathrm T}(y(t))Eg(y(t))\\ &-\eta^1 g^{\mathrm T}(y(t-\tau(t)))Eg(y(t-\tau(t)))\\ &+f^{\mathrm T}(x(t))Ff(x(t))\\ &-\eta^2 f^{\mathrm T}(x(t-z(t)))Ff(x(t-z(t))). \quad (17)\end{aligned}$$

By Lemma 1 and Assumption 1, the following inequalities hold,

$$\begin{aligned}&2x^{\mathrm T}(t)PBg(y(t-\tau(t)))\\ &\quad -\eta^1 g^{\mathrm T}(y(t-\tau(t)))Eg(y(t-\tau(t)))\\ &\quad \leqslant \frac{1}{\eta^1}x^{\mathrm T}(t)PBE^{-1}B^{\mathrm T}Px(t), \quad (18)\end{aligned}$$

$$\begin{aligned}&2y^{\mathrm T}(t)QDf(x(t-z(t)))\\ &\quad -\eta^2 f^{\mathrm T}(x(t-z(t)))Ff(x(t-z(t)))\\ &\quad \leqslant \frac{1}{\eta^2}y^{\mathrm T}(t)QDF^{-1}D^{\mathrm T}Qy(t), \quad (19)\end{aligned}$$

$$g^{\mathrm T}(y(t))Eg(y(t)) \leqslant y^{\mathrm T}(t)\Delta_g E\Delta_g y(t), \quad (20)$$

$$f^{\mathrm T}(x(t))Ff(x(t)) \leqslant x^{\mathrm T}(t)\Delta_f F\Delta_f x(t). \quad (21)$$

Substituting (18)—(21) into (17) yields

$$\begin{aligned}\dot V(x(t),y(t)) \leqslant & x^{\mathrm T}(t)\Big[-2PA\\ &+\frac{1}{\eta^1}PBE^{-1}B^{\mathrm T}P+\Delta_f F\Delta_f\Big]x(t)\\ &+y^{\mathrm T}(t)\Big[-2QC+\frac{1}{\eta^2}QDF^{-1}D^{\mathrm T}Q\\ &+\Delta_g E\Delta_g\Big]y(t). \quad (22)\end{aligned}$$

The remainder is in the similar way to the proof of Theorem 1. Therefore, if condition (15) holds, then the origin of system (14) is globally asymptotically stable, and independent of the magnitude of time varying delays.

When $\tau_{ij}(t)=\tau(t)\geqslant 0$ and $z_{ij}(t)=z(t)\geqslant 0$ in (2), (3) is replaced by the following expression:

$$\begin{aligned}\dot x(t) &= -Ax(t)+Bg(y(t-\tau(t))),\\ \dot y(t) &= -Cy(t)+Df(x(t-z(t))). \quad (23)\end{aligned}$$

Next, we will give the following stability result for system (23).

Theorem 2. Suppose that $\dot\tau(t)<1$ and $\dot z(t)<1$. If the following inequalities

$$\begin{aligned}&-2HC\Delta_g^{-1}+E+B^{\mathrm T}P(2PA)^{-1}PB/\eta^1\\ &\quad +HDF^{-1}D^{\mathrm T}H/\eta^2<0,\\ &-2LA\Delta_f^{-1}+F+D^{\mathrm T}Q(2QC)^{-1}QD/\eta^2\\ &\quad +LBE^{-1}B^{\mathrm T}L/\eta^1<0, \quad (24)\end{aligned}$$

have positive definite symmetric matrices P, Q, E, F, positive diagonal matrices $H=\mathrm{diag}(H_1,\cdots,H_m)$, $L=\mathrm{diag}(L_1,\cdots,L_n)$, then the origin of system (23) is globally asymptotically stable, and independent of the magnitude of time varying delay, where $\eta^1=1-\dot\tau(t)$, $\eta^2=1-\dot z(t)$.

Proof. Consider the following Lyapunov-Krasovskii functional,

$$\begin{aligned}V(x(t),y(t)) =& x^{\mathrm T}(t)Px(t)+y^{\mathrm T}(t)Qy(t)\\ &+2\sum_{j=1}^{m}\int_0^{y_j(t)}H_j g_j(s)\mathrm{d}s+2\sum_{j=1}^{n}\int_0^{x_j(t)}L_j f_j(s)\mathrm{d}s\\ &+\int_{t-\tau(t)}^{t}g^{\mathrm T}(y(s))(E_0+E)g(y(s))\mathrm{d}s\\ &+\int_{t-z(t)}^{t}f^{\mathrm T}(x(s))(F_0+F)f(x(s))\mathrm{d}s, \quad (25)\end{aligned}$$

where E_0, F_0 will be defined later. The derivative of (25) along the trajectories of (23) is

$$\begin{aligned}\dot V(x(t),y(t)) =& 2x^{\mathrm T}(t)P[-Ax(t)\\ &+Bg(y(t-\tau(t)))]\\ &+y^{\mathrm T}(t)Q[-Cy(t)+Df(x(t-z(t)))]\\ &+2g^{\mathrm T}(y(t))H[-Cy(t)+Df(x(t-z(t)))]\\ &+2f^{\mathrm T}(x(t))L[-Ax(t)+Bg(y(t-\tau(t)))]\\ &+g^{\mathrm T}(y(t))(E+E_0)g(y(t))\\ &-\eta^1 g^{\mathrm T}(y(t-\tau(t)))(E+E_0)g(y(t-\tau(t)))\\ &+f^{\mathrm T}(x(t))(F+F_0)f(x(t))\\ &-\eta^2 f^{\mathrm T}(x(t-z(t)))(F+F_0)f(x(t-z(t))). \quad (26)\end{aligned}$$

By Lemma 1 and Assumption 1, the following inequalities hold:

$$\begin{aligned}&-2x^{\mathrm T}(t)PAx(t)+2x^{\mathrm T}(t)PBg(y(t-\tau(t)))\\ &\quad \leqslant g^{\mathrm T}(y(t-\tau(t)))B^{\mathrm T}P(2PA)^{-1}\\ &\quad \cdot PBg(y(t-\tau(t))), \quad (27)\end{aligned}$$

$$\begin{aligned}&-2y^{\mathrm T}(t)QCy(t)+2y^{\mathrm T}(t)QDf(x(t-z(t)))\\ &\quad \leqslant f^{\mathrm T}(x(t-z(t)))D^{\mathrm T}Q(2QC)^{-1}\\ &\quad \cdot QDf(x(t-z(t))), \quad (28)\end{aligned}$$

$$\begin{aligned}&2g^{\mathrm T}(y(t))HDf(x(t-z(t)))\\ &\quad -\eta^2 f^{\mathrm T}(x(t-z(t)))Ff(x(t-z(t)))\\ &\quad \leqslant \frac{1}{\eta^2}g^{\mathrm T}(y(t))HDF^{-1}D^{\mathrm T}Hg(y(t)), \quad (29)\end{aligned}$$

$$2f^{\mathrm{T}}(x(t))LBg(y(t-\tau(t))) - \eta^1 g^{\mathrm{T}}(y(t-\tau(t)))Eg(y(t-\tau(t))) \leqslant \frac{1}{\eta^1}f^{\mathrm{T}}(x(t))LBE^{-1}B^{\mathrm{T}}Lf(x(t)). \quad (30)$$

Let

$$E_0 = \frac{1}{\eta^1}B^{\mathrm{T}}P(2PA)^{-1}PB,$$
$$F_0 = \frac{1}{\eta^2}D^{\mathrm{T}}H(2HC)^{-1}HD. \quad (31)$$

Then substituting (27)—(31) into (26) yields,

$$\dot V(x(t),y(t)) = g^{\mathrm{T}}(y(t))(-2HC\Delta_g^{-1} + E + B^{\mathrm{T}}P(2PA)^{-1}PB/\eta^1 + HDF^{-1}D^{\mathrm{T}}H/\eta^2)g(y(t)) + f^{\mathrm{T}}(x(t))(-2LA\Delta_f^{-1} + F + D^{\mathrm{T}}Q(2QC)^{-1}QD/\eta^2 + LBE^{-1}B^{\mathrm{T}}L/\eta^1)f(x(t)). \quad (32)$$

If condition (24) holds, then $\dot V(x(t),y(t))<0$ for $\forall x(t)\neq 0$ and $\forall y(t)\neq 0$. The rest is similar to the proof of Theorem 1. This completes the proof.

If we take another kind of processing method, we will have the following result for system (23).

Theorem 3. Suppose that $\dot\tau(t)<1$ and $\dot z(t)<1$. If the following inequalities

$$-2HC\Delta_g^{-1} - \Delta_g^{-1}QC\Delta_g^{-1} + E + B^{\mathrm{T}}P(2PA)^{-1}PB/\eta^1 + HDF^{-1}D^{\mathrm{T}}H/\eta^2 < 0,$$
$$-2LA\Delta_f^{-1} - \Delta_f^{-1}PA\Delta_f^{-1} + F + D^{\mathrm{T}}Q(QC)^{-1}QD/\eta^2 + LBE^{-1}B^{\mathrm{T}}L/\eta^1 < 0, \quad (33)$$

have positive diagonal matrices $P, Q, H = \mathrm{diag}(H_1, \cdots, H_m)$, $L = \mathrm{diag}(L_1, \cdots, L_n)$, positive definite symmetric matrices E, F, then the origin of system (23) is globally asymptotically stable, and independent of the magnitude of time varying delay, where $\eta^1 = 1-\dot\tau(t)$, $\eta^2 = 1-\dot z(t)$.

Proof. Consider the Lyapunov-Krasovskii functional (25), where E_0, F_0 will be defined later. The derivative of (25) along the trajectories of (23) is the same as (26).

By Lemma 1 and Assumption 1, the following inequalities hold:

$$-x^{\mathrm{T}}(t)PAx(t) + 2x^{\mathrm{T}}(t)PBg(y(t-\tau(t))) \leqslant g^{\mathrm{T}}(y(t-\tau(t)))B^{\mathrm{T}}P(PA)^{-1}\cdot PBg(y(t-\tau(t))), \quad (34)$$

$$-y^{\mathrm{T}}(t)QCy(t) + 2y^{\mathrm{T}}(t)QDf(x(t-z(t))) \leqslant f^{\mathrm{T}}(x(t-z(t)))D^{\mathrm{T}}Q(QC)^{-1}\cdot QDf(x(t-z(t))), \quad (35)$$

$$2g^{\mathrm{T}}(y(t))HDf(x(t-z(t))) - \eta^2 f^{\mathrm{T}}(x(t-z(t)))Ff(x(t-z(t))) \leqslant \frac{1}{\eta^2}g^{\mathrm{T}}(y(t))HDF^{-1}D^{\mathrm{T}}Hg(y(t)), \quad (36)$$

$$2f^{\mathrm{T}}(x(t))LBg(y(t-\tau(t))) - \eta^1 g^{\mathrm{T}}(y(t-\tau(t)))Eg(y(t-\tau(t))) \leqslant \frac{1}{\eta^1}f^{\mathrm{T}}(x(t))LBE^{-1}B^{\mathrm{T}}Lf(x(t)), \quad (37)$$

$$-x^{\mathrm{T}}(t)PAx(t) \leqslant -f^{\mathrm{T}}(x(t))\Delta_f^{-1}PA\Delta_f^{-1}f(x(t)), \quad (38)$$

$$-y^{\mathrm{T}}(t)QCy(t) \leqslant -g^{\mathrm{T}}(y(t))\Delta_g^{-1}QC\Delta_g^{-1}g(y(t)). \quad (39)$$

Let

$$E_0 = \frac{1}{\eta^1}B^{\mathrm{T}}P(PA)^{-1}PB,$$
$$F_0 = \frac{1}{\eta^2}D^{\mathrm{T}}H(HC)^{-1}HD. \quad (40)$$

Then substituting (34)—(40) into (26) yields,

$$\dot V(x(t),y(t)) = g^{\mathrm{T}}(y(t))(-2HC\Delta_g^{-1} - \Delta_g^{-1}QC\Delta_g^{-1} + E + B^{\mathrm{T}}P(PA)^{-1}PB/\eta^1 + HDF^{-1}D^{\mathrm{T}}H/\eta^2)g(y(t)) + f^{\mathrm{T}}(x(t))(-2LA\Delta_f^{-1} - \Delta_f^{-1}PA\Delta_f^{-1} + F + D^{\mathrm{T}}Q(QC)^{-1}QD/\eta^2 + LBE^{-1}B^{\mathrm{T}}L/\eta^1)f(x(t)). \quad (41)$$

If condition (33) holds, then $\dot V(x(t),y(t))<0$ for $\forall x(t)\neq 0$ and $\forall y(t)\neq 0$. The rest is similar to the proof of Theorem 1. This completes the proof.

Remark 1. Theorem 1 has a wider range of application than Corollary 1, Theorem 2 and Theorem 3 because model (1) is more general than models (14) and (23). That is to say, Corollary 1, Theorem 2 and Theorem 3 are not suitable for model (1), while Theorem 1 can be applied to models (14) and (23).

Remark 2. Model (1) with constant delays has also been studied in Refs. [3,7]. Theorem 1 in Ref. [3] provides a simple criterion to guarantee the global asymptotic stability for model (1), but the sign difference of entries in connection matrix is ignored and the effects of neuron excitatory and inhibitory on neural networks is not considered. Theorem 2 in Ref.

[7] gives a sufficient condition guaranteeing the global exponential stability of model (1) based on an algebraic inequality. Although the suitability of the criterion in Ref. [7] is improved due to involving many parameters to be tuned, it is not easy to verify this criterion by efficiently choosing unknown parameters because we have no a systematic approach to tune those parameters in advance. Theorem 1 in the present paper is obtained via linear matrix inequality (LMI) technique, therefore, it is easy to be verified and the sign difference of connection matrix is eliminated. Therefore, Theorem 1 in the present paper overcomes the shortcomings of the results in Refs. [3, 7]. Some sufficient conditions guaranteeing the global asymptotic stability of model (23) with constant delay are derived from Refs. [5, 6, 8, 9] based on some algebraic inequality techniques. However, the obtained results in the present paper generalize and improve the results in Refs. [5, 6, 8, 9] in the aspects of complexity of network model, verification of the stability result and neuron excitatory and inhibitory on neural networks.

Remark 3. Model (23) with constant delay is also studied in Ref. [4]. If we let $\boldsymbol{P}=\boldsymbol{Q}=\boldsymbol{E}=\boldsymbol{F}=\boldsymbol{H}=\boldsymbol{L}=\boldsymbol{I}$ in Theorem 2, then (24) is just the result of Theorem 1 and Theorem 2 in Ref. [4]. Therefore, the results in Ref. [4] are special cases of our result Theorem 2.

2 Numerical example

Consider the following bi-directional associative memory neural networks with constant delays:

$$\begin{aligned}\dot{x}(t) &= -\boldsymbol{A}x(t)+\boldsymbol{B}g(y(t-\tau))+\boldsymbol{U},\\ \dot{y}(t) &= -\boldsymbol{C}y(t)+\boldsymbol{D}f(x(t-z))+\boldsymbol{J},\end{aligned} \quad (42)$$

where $\boldsymbol{A}=\boldsymbol{C}=2\boldsymbol{I}$, $\boldsymbol{B}=\boldsymbol{D}=\begin{bmatrix} c & c \\ c & -c \end{bmatrix}$, $g_j(u_j)=f_j(u_j)=0.5(|u_j+1|-|u_j-1|)$, $\boldsymbol{U}, \boldsymbol{J}$ are real column vectors with 2×1 dimensions, $\tau>0$, $z>0$ are any bounded and constant delays, respectively. If we choose $\boldsymbol{P}=\boldsymbol{Q}=\boldsymbol{I}, \boldsymbol{E}_i=\boldsymbol{F}_i=\boldsymbol{I}, i=1,2$, in Theorem 1 of the present paper, then in order to guarantee the hold of Theorem 1, c must satisfy $|c|<1$. Pertaining to this example, Theorem 1 in Ref. [3] holds if and only if $|c|<1$. Similarly, the results in Refs. [5,6] also require $|c|<1$. Obviously, Theorem 1 in the present paper and Theorem 1 in Ref. [3], results in Refs. [5,6] provide different sufficient conditions for the stability of the BAM system defined by (42). In Theorem 3 of the present paper, if we choose $\boldsymbol{P}=\boldsymbol{Q}=\boldsymbol{E}=\boldsymbol{F}=\boldsymbol{H}=\boldsymbol{L}=\boldsymbol{I}$, then we have $|c|<\sqrt{5/3}$. Thus, it can be conclude that if $|c|\geqslant 1$, the results in Refs. [3,5,6] are not satisfied, whereas the conditions of Theorem 3 in the paper still hold for $1\leqslant |c|<\sqrt{5/3}$.

3 Conclusions

The principle contribution of this paper is the results that ensure the global asymptotic stability of BAM neural networks with time varying delays. The obtained results establish a relationship between the network parameters of neural networks independent of the magnitude of time varying delays. Further, the obtained results possess the structure of linear matrix inequality (LMI), so they can be easily verified. A comparison between the obtained results and the previous results has also been made to show the effectiveness of the obtained results.

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